Nernst effect in the phase-fluctuating superconductor InO$_x$

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We present a study of the Nernst effect in amorphous 2D superconductor InO$_x$, whose low carrier density implies low phase rigidity and strong superconducting phase fluctuations. Instead of presenting the abrupt jump expected at a BCS transition, the Nernst signal evolves continuously through the superconducting transition as previously observed in underdoped cuprates. This contrasts with the case of Nb$_{0.15}$Si$_{0.85}$, where the Nernst signal due to vortices below T$_c$ and by Gaussian fluctuations above are clearly distinct. The behavior of the ghost critical field in InO$_x$ points to a correlation length which does not diverge at T$_c$, a temperature below which the amplitude fluctuations freeze, but phase fluctuations survive.

Those past years have witnessed the emergence of the Nernst effect as an important probe of Superconducting Fluctuations (SF), following the observation of an anomalous Nernst signal above T$_c$ in cuprates [1]. In amorphous superconducting thin films of Nb$_{0.15}$Si$_{0.85}$, a Nernst signal produced by Cooper-pair fluctuations could be detected in a wide temperature and field range [2, 3]. Close to T$_c$, the magnitude of the Nernst coefficient found in this experiment was in very good agreement with the predictions of a theory by Ussishkin, Sondhi and Huse (USH) for the transverse thermoelectric response of the Gaussian fluctuations of the Superconducting Order Parameter(SOP) [4]. This is not the case of underdoped cuprates, where the Nernst signal does not follow the predictions of the USH theory[4] and phase fluctuations of the SOP are believed to play a major role.

To address this issue, new theories have been proposed addressing cases where the Nernst signal is only generated by phase fluctuations of the SOP [5] or by quantum fluctuations near a Superconductor-Insulator Transition (SIT) [6]. On the experimental side, recent measurements on organic quasi-2D superconductors [7] detected a finite Nernst signal above T$_c$ in a temperature range widening with the approach of the Mott insulator as in the case of cuprates [1]. However, since the Nernst response of normal electrons scales with their mobility [8], the normal-state Nernst response is not negligible in either cuprate or organic superconductors. This complicates any quantitative comparison of the measured Nernst signal with theoretical predictions.

In this Letter, we report on the case of InO$_x$. Several factors make thin films of this system an appealing candidate for the study of the Nernst signal generated by superconducting phase fluctuations. First of all, due to its low carrier density, a poor superfluid stiffness and, consequently, strong phase fluctuations are expected [10]. Moreover, the normal state is a simple dirty metal, with a negligible Nernst response. This system is also believed to host a Kosterlitz-Thouless-Berezinskii (KTB) transition [11]. Finally, due to its large sheet resistance, quan-
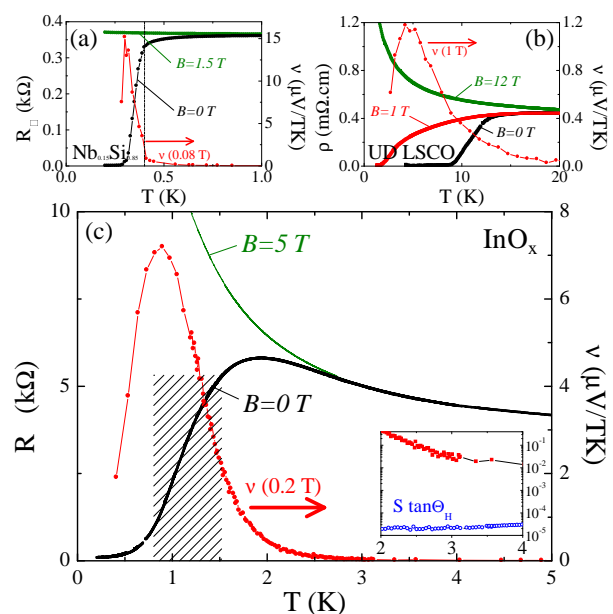

FIG. 1: The effect of superconducting transition on resistance and Nernst signal in Nb$_{0.15}$Si$_{0.85}$ [2](panel *a*), La$_{1.94}$Sr$_{0.06}$CuO$_4$ single crystal [9](panel *b*) and InO$_x$ (panel *c*). The shaded region represents the temperature range corresponding to 0.1-0.9 of $R_\square^{max}$. The inset of panel *c* compares the Nernst signal with $S \tan \theta_H$. Note the sharp increase of the Nernst signal at T$_c$ for Nb$_{0.15}$Si$_{0.85}$, in contrast to continuous change observed for LSCO and InO$_x$.

tum fluctuations of the phase of the SOP are expected to give rise to a SIT at zero-temperature.

According to our findings, the Nernst effect in this system shares common features with cuprates. In contrast with Nb$_{0.15}$Si$_{0.85}$, its temperature dependence does not follow the predictions of USH theory. Moreover, both the field and temperature dependence of the Nernst signal in InO$_x$ indicate that the blurred transition reflects a regime of superconducting fluctuations whose

Correlation Length(CL) does not diverge. Our analysis is based on the previous study of the Nernst data in $Nb_{0.15}Si_{0.85}$ [2, 3], which established the link between the Nernst signal and the CL [3].

The 300 Å-thick amorphous InO_x film used in this study is deposited on a glass substrate by e-gun evaporation of In_2O_3 in oxygen atmosphere [12]. Using a one-heater-two-thermometers setup, four point resistance, Hall effect and thermoelectric measurements are measured in a single cool-down. The as-prepared film is insulating down to the lowest measured temperature of 60 mK. After thermal annealing at 50°C under vacuum as described elsewhere [13], the sheet resistance decreases by about 30 % and a superconducting state appears. According to optical absorption experiments, this drop of resistivity is the consequence of the volume shrinkage of the sample during annealing [12]. During all measurements, the film has been kept below liquid nitrogen temperature to avoid aging effects.

Fig. 1 compares the behavior of the Nernst signal $N = E_y/(-\nabla_x T)$, measured in the low field limit, in the vicinity of the superconducting transition in three different systems. In the case of $Nb_{0.15}Si_{0.85}$ [2], N increases abruptly at the BCS superconducting transition. It was shown [2, 3] that above T_c, the Nernst signal is generated by Cooper pairs fluctuations, and below T_c, by well defined vortices. In contrast, in $La_{1.94}Sr_{0.06}CuO_4$ [9], as seen in Fig. 1b, no distinct anomaly in $N(T)$ is visible at any temperature separating these two regimes. As seen in Fig. 1c, the same is true for InO_x: The Nernst signal evolves continuously across the superconducting transition. As seen in the inset of the same figure, the signal is at least 100 times larger than the product of the Seebeck coefficient and the Hall angle. Since the latter ($S \tan\theta$) sets the order of magnitude of the normal-state response, the observed Nernst signal is almost entirely due to SF.

A low carrier density is one fundamental feature shared by InO_x and $La_{1.94}Sr_{0.06}CuO_4$. The Hall coefficient measured in our film ($R_H = 6.10^{-9}$ $m^3.C^{-1}$) is close to the one found in $La_{1-x}Sr_xCuO_4$ (LSCO) at x=0.05 [14] and yields a carrier density of $n = 10^{21}$ cm^{-3}. On the other hand, the Hall coefficient in $Nb_{0.15}Si_{0.85}$ is 80 times lower [2], implying a much higher carrier density. Since the superfluid stiffness is proportional to the superfluid density, superconductors such as InO_x and $La_{1.94}Sr_{0.06}CuO_4$ are expected to display strong phase-fluctuations [10]. This is the most plausible source of this peculiar Nernst response in the vicinity of the superconducting transition.

Since $\tan\theta_H \ll 1$,the Nernst and resistivity data suffice to determine the transverse Peltier response $\alpha_{xy} = N/R_\square$ as presented in Figure 2. Above T_c, for short-lived Cooper pairs described as Gaussian fluctuations of the SOP, this coefficient is simply related to the superconducting CL, $\alpha_{xy}/B \propto \xi^2$ at 2D [4]. The inset of Figure 2 shows that in $Nb_{0.15}Si_{0.85}$ cooling leads to a

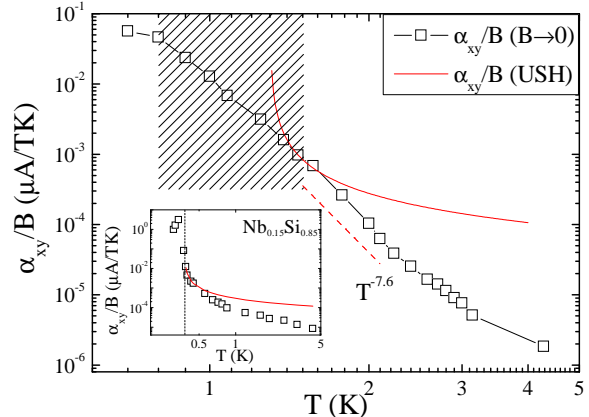FIG. 2: Transverse Peltier coefficient α_{xy}/B versus temperature for $B \rightarrow 0$. The shaded region represents the temperature range where T_c is expected. The theoretical prediction of USH theory is represented by the red line. The inset shows data for $Nb_{0.15}Si_{0.85}$ along with the USH prediction (from [2]). In this compound, $T_c = 0.38$ K, is represented by the vertical line.

steep increase in $(\frac{\alpha_{xy}}{B})_{B \rightarrow 0}$ at T_c, indicating the divergence of the CL. In contrast, for InO_x, α_{xy}/B evolves continuously and no abrupt change is observed on the whole temperature range of measurements; i.e; 0.6 K to 4.5 K. This suggests that there is no diverging CL and therefore, no true phase transition at T_c, the temperature corresponding to the formation of Cooper pairs, expected to be located in the $0.8 - 1.5$ K range and represented by shaded regions in figures 1 and 2.

We now proceed to an analysis of the field dependence of the Nernst data, which leads to the same conclusion. Fig. 3a shows that, for each temperature, the Nernst signal $N(B)$ peaks with a maximum at a temperature-dependent magnetic field scale $B^*(T)$. This peak can be clearly observed in $N(B)$ down to a temperature of 0.9 K. As discussed in previous studies on $Nb_{0.15}Si_{0.85}$ [2, 3], at any temperature and magnetic field, α_{xy}/B depends only on the size of SF. At zero magnetic field, this size is set by the CL. At high field, this size is set by the magnetic length $l_B = (\hbar/2eB)^{1/2}$ when it becomes shorter than the zero-field CL. Thus, this coefficient acquires a characteristic field-temperature dependence that is observed in $Nb_{0.15}Si_{0.85}$ [3] and in InO_x as shown Fig. 3b. α_{xy}^{sc}/B is field-independent at low magnetic field, however, at high magnetic field, all the data evolve toward a single curve weakly dependent on temperature. This crossover is responsible for the peak observed at $B^*(T)$ in the field dependence of the Nernst signal $N(B)$ (see arrows in Fig. 3a).
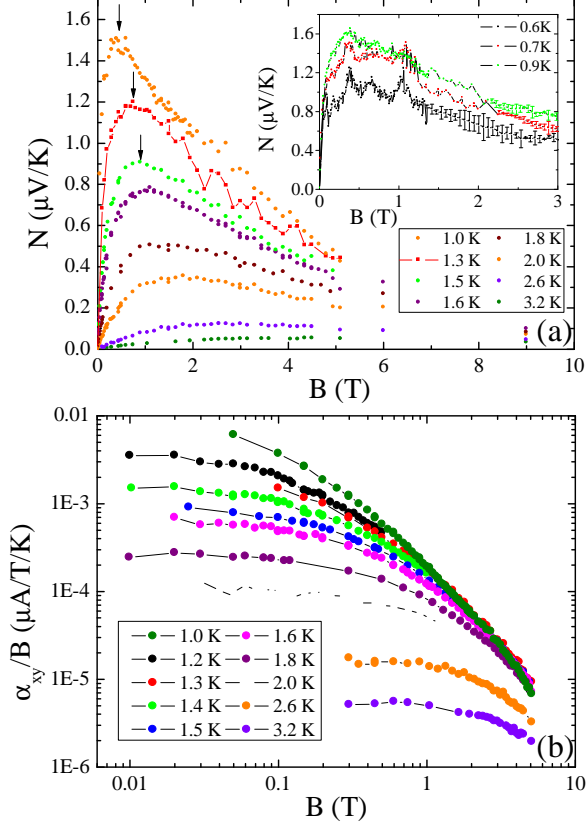
The temperature dependence of $B^*(T)$ is presented in

FIG. 3: a) Nernst signal as a function of magnetic field for different temperatures $T \geq 1\ K$. The inset shows the low temperature data, $T < 1\ K$. Arrows indicate the ghost critical field B^*. b) α_{xy}/B versus magnetic field in the regime of superconducting fluctuations, $T > 1\ K$

fig. 4b for InO_x (main panel) and for $Nb_{0.15}Si_{0.85}$ [3] (inset). In both systems, above T_c, $B^* \propto \ln T$ as expected for the Ghost Critical Field (GCF), $\Phi_0/2\pi\xi^2$, set by the BCS CL, $\xi = \xi_0\varepsilon^{-1/2}$ where $\varepsilon = \ln T/T_c$. In the case of $Nb_{0.15}Si_{0.85}$, ξ_0 and T_c could be independently determined and compared with the GCF determined from the Nernst data. For InO_x, on the other hand, we set T_c and ξ_0 such that the GCF line fits $B^*(T)$. The thick line Fig. 4 is a fit using $\xi_0 = 8.4\pm0.2$ nm and $T_c = 1.2$ K. This value of T_c corresponds to the mid-point of the resistive transition, as seen Figure 1. A similar conclusion on the position of T_c was drawn by a recent study on InO_x[11].

With the temperature dependence of the CL just determined, we find that USH formula[4], when $B \to 0$:

$$\alpha_{xy}^{sc} = \frac{1}{12\pi} \frac{k_B e}{\hbar} \frac{\xi^2}{l_B^2} \qquad (1)$$

is close to the measured α_{xy}, as seen Fig. 2. However,

α_{xy} decreases with temperature as fast as $T^{-7.6}$, much faster than predicted by the USH theory. One possibility is that the CL is too short for the applicability of the USH theory on a large temperature range. Indeed, in $Nb_{0.15}Si_{0.85}$, where ξ_0 is larger, α_{xy}/B was found to deviate from USH theory for $T > 1.3 * T_c$, see inset of Fig.2. Since ξ_0 is shorter in InO_x, the deviation from theory is expected to occur closer to T_c. Another possibility is a deep difference in the nature of fluctuations in the two systems. A recent model of phase-only fluctuations [5] predicts a faster decrease of the Nernst signal above T_{KTB} compared to what is expected in the Gaussian picture in the temperature range above T_{BCS}, in qualitative agreement with what is seen here. However, if the fast decrease of the Nernst signal observed up to 4 K is due to fluctuations with frozen amplitude, it would imply T_{BCS} to be above 4 K, which is unlikely.

While we find difficult to draw conclusions from temperature dependence of the Nernst data, the interpretation of the field position of the Nernst peak as the GCF appears straightforward. According to our analysis, this field scale reflects the CL, no matter the precise nature of SF, Gaussian or phase-only. This recently received some theoretical support. Functional forms for the field dependence compatible with a maximum at the GCF have been predicted by a theory expanding the USH theory to finite field [15] and by a recent theory of the Nernst effect in the vicinity of a SIT [6].

In $Nb_{0.15}Si_{0.85}$ (see inset of Fig. 4b) the GCF vanishes and at T_c (reflecting the divergence of the CL) mirrors the behavior of the $H_{c2}(T)$ below T_c. One striking observation of this work is the breakdown of this picture in InO_x . As seen in Fig. 3a, $B^*(T)$ keeps decreasing down to 0.9 K, well below our estimate of $T_c = 1.2$ K. This indicates that the CL does not diverge and that no true phase transition occurs at T_c, the temperature associated with the formation of Cooper pairs. An identical conclusion was drawn from the temperature dependence of α_{xy}/B where no abrupt change is observed upon crossing T_c. This leads us to conclude that the wide superconducting transition is not simply the consequence of a large critical region, or sample inhomogeneity, but reflects the presence of an intermediate fluctuation region between T_c, where amplitude fluctuations freeze, and a lower temperature, T_{KTB}, where phase coherence should be established. Such a region of phase-only fluctuations in InO_x was recently inferred from high-frequency conductivity measurements [11].

Upon cooling, phase fluctuations are expected to disappear at KTB transition where the vortex and anti-vortex bind together. Since the vortices are a major source of the Nernst signal, the latter should be strongly affected by KTB transition. Below 0.9 K, the overall magnitude of the Nernst signal decreases and the field dependence $N(B)$ displays a broad maximum with complicated but reproducible substructures. The reduced amplitude
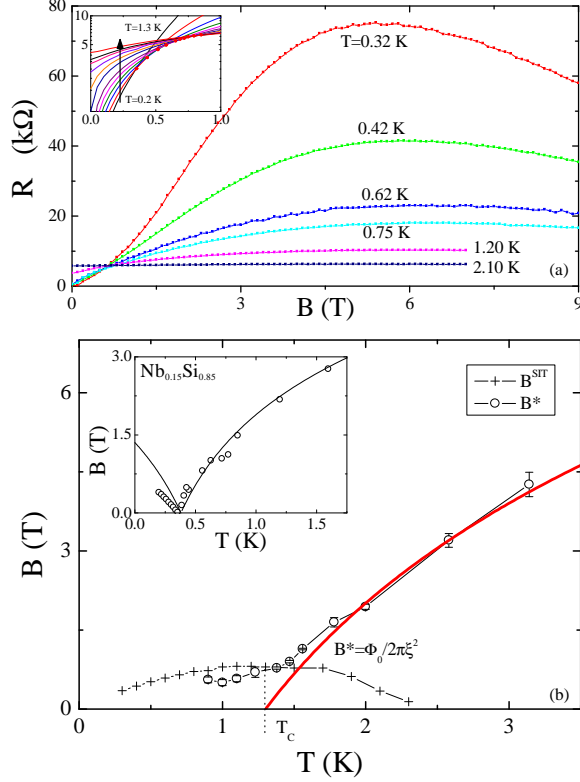
tance curves, thus providing support to this interpretation of the negative magnetoresistance.

In contrast to the SIT observed in $Nb_{0.15}Si_{0.85}$ [19], MoGe [20], or Bi [21] thin films, the crossing-point $B_{SIT}(T)$, reported Fig. 4, is temperature dependent. This behavior has been discussed previously for this compound [22] and remains yet to be understood.

To summarize, measuring Nernst signal and resistivity in InO_x, we found that the transverse Peltier coefficient evolves continuously across the superconducting transition. Furthermore, we find that the GCF keeps decreasing on the temperature range where the Cooper pair formation is expected to occur. This indicates that no true phase transition occurs at T_c and implies the existence of a regime of phase-only fluctuations of the SOP. The similarity between the temperature dependence of the Nernst signal measured in InO_x and the underdoped cuprates is additional support for the existence of a regime of phase-only fluctuations in the latter system.

FIG. 4: a) Magnetoresistance measured for several temperatures. The inset focus on the low field part showing the crossing between adjacent isotherms. b) Phase diagram representing B_{SIT} and the position of the Nernst maximum B^*. The thick line is an adjustment of the ghost critical field $\Phi_0/2\pi\xi^2$. The inset shows B^* measured in $Nb_{0.15}Si_{0.85}$ [3]. The line below T_c represents $H_{c2}(T)$.

points to a reduced vortex mobility below 0.9 K, a possible signature of the KTB transition. The multiple peaks observed in $N(B)$ are reminiscent of what was also observed in the field dependence of the Nernst signal in hole-doped cuprates at low temperatures and tentatively attributed to a plastic flow of vortices [1].

This behavior may also be related to the SIT and Bose insulating properties of this system. As previously observed in InO_x [16, 17], at low temperature, the magnetoresistance increases steeply following the magnetic-field induced SIT, (see Fig. 4a). According to the dirty-boson model [18], the insulating side is formed of Cooper pairs localized by the quantum melting of the vortex system. Within this framework, the negative magnetoresistance observed at high field is due to the pair-breaking effect of the magnetic field when $H > H_{c2}$. Using our previous estimation of the CL, we find $H_{c2} = 4.7 \pm 0.2T$, which is about the position of the maximum of the magnetoresis-

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